The derivative of (7) has, by virtue of the equations of perturbed motion, the form $V^{\cdot} = \omega x_2^{\cdot} V_3 / l_2 = 0$, and this is correct since $V_3 = 0$. Therefore, on the basis of the Rumiantsev theorem [4] the inequality (8) is a sufficient condition of stability of the unperturbed motion (2) with respect to the variables $p - \omega\gamma_1$, q, r, γ_2 and γ_3.

The unstable permanent rotations (2) can be separated out by considering the linearized system of equations of perturbed motion

$$x_1^{\cdot} = (1 - \delta)\,\omega\,(l_3 x_2 + l_2 x_3) - u_3^{\circ} y_2 - l_2 u_{33}^{\circ} y_3, \quad x_2^{\cdot} = (\delta - 1)\,\omega\,(l_3 x_1 + l_1 x_3) + u_3^{\circ} y_1, \quad y_1^{\cdot} = -l_3 x_2 + l_2 x_3 + \omega\,(l_3 y_2 - l_2 y_3), \quad y_2^{\cdot} = l_3 x_1 - \omega l_3 y_1, \quad y_3^{\cdot} = -l_2 x_1 + \omega l_2 y_1, \quad x_3^{\cdot} = 0 \tag{9}$$

The characteristic equation of (9) has the form

$$\sigma^4 (\sigma^2 + g_0) = 0, \quad g_0 = \omega^2 [1 + (1 - \delta)^2 l_3^2] - u_{33}^{\circ} l_2^{\circ} + 2 u_3^{\circ} l_3 \tag{10}$$

It is clear that when $g_0 < 0$, one of the roots of (10) is positive and the motion (2) in its first approximation will, by the Liapunov theorem on stability, be unstable.

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CERTAIN PARTICULAR CASES OF STABILITY IN FIRST APPROXIMATION OF DIFFERENCE SYSTEMS

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The results of this paper can be regarded as a transposition of the results of Chetaev obtained for the finite systems of differential equations [1] to the denumerable systems of the finite difference equations. We use the concepts of [2].

Let us consider the system

$$y_s(m+1) = \sum_{i=1}^{\infty} p_{si}(m)\, y_i(m), \quad m = 0, 1, \ldots \tag{1}$$

Here and henceforth $s = 1, 2, \ldots$, the functions p_{si} are bounded and the series $|p_{s1}(m)| + |p_{s2}(m)| + \ldots$ converge uniformly in m for $0 \leqslant m < \infty$. We define $\| y(m) \| = \sup_s | y_s(m) |$.

We use the following system with constant coefficients:

$$x_s(m+1) = \sum_{i=1}^{\infty} c_{si} x_i(m) \qquad (2)$$

as the approximate system for solving the problem of stability of (1). We set

$$L = \sup_s \sum_{i=1}^{\infty} |c_{si}| \qquad (3)$$

We shall say that the zero (unperturbed) solution $x_s(m) \equiv 0$ of system (2) is exponentially stable if all perturbed solutions of this system obey for all $m \geqslant m_0$ the law [3]

$$\| x(m) \| \leqslant B \| x(m_0) \| \exp[-\alpha(m - m_0)] \qquad (4)$$

where $B \geqslant 1$ and $\alpha > 0$ are independent of m_0 and unaffected by the choice of $x_s(m_0)$ from the region $\| x(m_0) \| < \varepsilon$, where ε is sufficiently small. We shall say in a similar manner of the solutions of the system (1).

T h e o r e m 1. Let the system (1) and system (2) with constant coefficients be both given, and connected by the relation

$$\sup_s \sup_m \left\{ \sum_{i=1}^{\infty} |p_{si}(m) - c_{si}|, \quad m_0 \leqslant m \leqslant \infty, \quad s = 1, 2, \ldots \right\} \leqslant M < \infty \qquad (5)$$

If the zero solution of (2) is exponentially stable, then for sufficiently small M the zero solution of (1) will also be exponentially stable.

P r o o f. Let

$$T = \alpha^{-1} \ln 4B + \tau \; (\tau \geqslant 0) \qquad (6)$$

$$\delta = \varepsilon / (2B) \qquad (7)$$

where α and B are quantities given by (4) and ε is an arbitrary positive number for which the inequality (4) holds.

We consider the solutions $y_s(m)$ and $x_s(m)$ of (1) and (2), determined by the initial conditions

$$\| y(m_0) \| = \| x(m_0) \| < \delta \qquad (8)$$

From (4), taking into account (8) and (7) we obtain

$$\| x(m) \| < \varepsilon/2 \quad \text{for} \quad m \geqslant m_0 \qquad (9)$$

Further, taking into account (8) and (6) and setting $m = m_0 + T$, we obtain from (4)

$$\| x(m_0 + T) \| < \delta/4 \qquad (10)$$

Next we shall prove the inequality

$$\Delta_m \equiv \| y(m) - x(m) \| < \begin{cases} M\delta B[(L+M)^{m-m_0} - 1]/(L+M-1) & \text{when } L+M \neq 1 \\ M\delta B(m-m_0) & \text{when } L+M = 1 \end{cases} \qquad (11)$$

We write (1) in the form

$$y_s(m+1) = \sum_{i=1}^{\infty} c_{si} y_i(m) + \sum_{i=1}^{\infty} [p_{si}(m) - c_{si}] y_i(m) \qquad (12)$$

From (12) and (2), using (3) and (5) we obtain $\Delta_{m+1} \leqslant L\Delta_m + M \| y(m) \|$. Since $\| x(m) \| \leqslant \Delta_m + \| y(m) \|$, taking (4) and (8) into account, we find that $\Delta_{m+1} \leqslant (L + M)\Delta_m + M\delta B$. Replacing m in this inequality by $m_0, m_0 + 1, m_0 + 2, \ldots, m - 1$

and taking (8) into account, we obtain the inequality (11).

Obviously, we can choose M in (11) such that

$$\Delta_m < \delta/4 \quad \text{for} \quad m_0 \leqslant m \leqslant m_0 + T \tag{13}$$

Further, using (13), (9), (7) and (10), we obtain

$$\|y(m)\| < \varepsilon \quad \text{for} \quad m_0 \leqslant m \leqslant m_0 + T$$
$$\|y(m_0 + T)\| < \delta/2$$

We take $m_0 + T$ as the initial value and consider the solutions of (1) and (2) determined by the initial conditions

$$\|y(m_0 + T)\| = \|x(m_0 + T)\| < \delta/2$$

and repeat all the above arguments beginning with the inequality (8). Let us assume that

$$\|y(m)\| < \varepsilon / 2^{n-1} \quad \text{for} \quad m_0 + (n-1)\,T \leqslant m \leqslant m_0 + nT$$

and

$$\|y(m_0 + nT)\| < \delta/2^n$$

We shall show that in this case we have

$$\|y(m)\| < \varepsilon/2^n \quad \text{for} \quad m_0 + nT \leqslant m \leqslant m_0 + (n+1)\,T \tag{14}$$

$$\|y[m_0 + (n+1)\,T]\| < \delta/2^{n+1} \tag{15}$$

Taking $m_0 + nT$ as the initial value, we consider the solutions of (1) and (2) determined by the initial conditions

$$\|y(\overline{m}_0)\| = \|x(\overline{m}_0)\| < \delta / 2^n \; (\overline{m}_0 = m_0 + nT)$$

From (4) with $m \geqslant \overline{m}_0$ and $\|x(\overline{m}_0)\| < \delta / 2^n < \varepsilon / (B2^{n+1})$ we obtain

$$\|x(m)\| < \varepsilon/2^{n+1} \quad \text{for} \quad m \geqslant \overline{m}_0 \tag{16}$$

after which, from (4) with $m = m_0 + T$ and $\|x(\overline{m}_0)\| < \delta/2^n$ with (6) taken into account, we obtain

$$\|x(\overline{m}_0 + T)\| < \delta/2^{n+2} \tag{17}$$

The inequality (13) then becomes

$$\Delta_m < \delta/2^{n+2} \quad \text{for} \quad m \geqslant \overline{m}_0 \tag{18}$$

Using (18), (16) and (7) we obtain

$$\|y(m)\| < \varepsilon/2^n \quad \text{for} \quad \overline{m}_0 \leqslant m \leqslant \overline{m}_0 + T \tag{19}$$

Further, using (18) with $m = \overline{m}_0 + T$ and (17), we obtain

$$\|y(\overline{m}_0 + T)\| < \delta/2^{n+1} \tag{20}$$

Substituting into (19) and (20) the value $\overline{m}_0 = m_0 + nT$, we obtain the inequalities (14) and (15), and from (14) we obtain

$$\|y(m)\| < 2\varepsilon \exp\left[-(m - m_0) \ln 2 / (\alpha^{-1} \ln 4B + \tau)\right]$$

which proves the theorem.

We consider, in addition to the systems (1) and (12), the system

$$y_s(m+1) = \sum_{i=1}^{\infty} p_{si}(m)\, y_i(m) + R_s[m, y_i(m)] \tag{21}$$

$$(\|R(m, y_i)\| < \gamma \|y\|, \quad i = 1, 2, \ldots)$$

in a region $D: \| y(m) \| \leqslant H, \ m = 0, 1, \ldots \ (H = \text{const})$.

Theorem 2. Let the systems (21) and (2) be connected by a relation of the type (5). If the zero solution of the system (2) is exponentially stable, then for sufficiently small γ and M the zero solution of the system (21) will also be exponentially stable.

The proof of Theorem 2 differs from that of Theorem 1 only in the fact that M in the inequality (11) is replaced by $M + \gamma$.

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ON EQUIVALENCE OF THE EQUATIONS OF MOTION OF NONHOLONOMIC SYSTEMS

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We prove the equivalence of the equations of motion of nonholonomic systems with constraints linear in velocities, obtained by various methods. At present, the equations of motion of nonholonomic systems exist in various forms. Naturally, the question of their identity to each other was brought up in [1 — 3], and the problem was also discussed in [4 — 8] and in the dissertation of M. I. Efimov (*).

1. The author of [1 — 3] postulates that the final form of the equations of motion of a system obtained by transforming the general dynamic equations depends on the point at which the equations of nonholonomic constraints are taken into account. He states that in the general case of arbitrary nonholonomic systems with constraints which are linear in velocities, the equations constructed by different methods cannot be guaranteed to be identical. Volterra [9], Appell [10] and MacMillan [11] derive the equations of motion from the general dynamic equation in Cartesian coordinates and bring the nonholonomic constraints into the discussion at once. Hamel [12], Chaplygin [13] and Voronets [14] bring in the nonholonomic constraints after the general dynamic equations have been transformed to the generalized coordinates. In the opinion of the author of [1—3], the equations of motion obtained using the methods of Volterra, Appell and MacMillan on one hand, and the methods of Voronets (Chaplygin) and Hamel on the other hand, will not, in general, be identical, i.e. the systems of equations will not be equivalent to each

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